

THURSDAY, OCTOBER 9, 1919.

MATHEMATICAL TEXT-BOOKS.

- (1) *Empirical Formulas*. By Prof. Theodore R. Running. (Mathematical Monographs, No. 19.) Pp. 144. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1917.) Price 7s. net.
- (2) *Differential and Integral Calculus*. By Dr. H. B. Phillips. *Differential*. Pp. v+162. *Integral*. Pp. v+194. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1916-17.) Price 9s. 6d. net.
- (3) *A First Course in the Calculus*. Part i. *Powers of X*. By Dr. W. P. Milne and G. J. B. Westcott. (Bell's Mathematical Series.) Pp. xx+196. (London: G. Bell and Sons, Ltd., 1918.) Price 3s. 6d.
- (4) *Dynamics*. Part II. By R. C. Fawdry. (Bell's Mathematical Series.) Pp. viii+179-355+vii. (London: G. Bell and Sons, Ltd., 1919.) Price 2s. 6d.
- (5) *Solid Geometry, including the Mensuration of Surfaces and Solids*. By Prof. R. S. Heath. Fourth edition. Pp. iv+123. (London: Rivingtons, 1919.) Price 4s.

(1) ONE of the most important activities of the practical, as well as of the theoretical, man of science is the discovery of laws. Given a number of observations, the problem is to correlate them in the form of a single analytical expression. The basis of such discovery is the recognition of a curve as being one the equation of which is known. But, strictly speaking, there is only one curve that is really recognisable, and this is the straight line. A piece of a circle can easily be mistaken for a piece of an ellipse, and a parabola for a catenary; but if a sufficiently long piece of a curve is straight, then the curve can be pronounced to be a straight line. If, then, it is possible to plot the results of observation in such a way that the resulting points lie on a straight line (even if there are some casual, experimental errors and consequent deviations), then we can at once deduce the law.

This fact underlies the major part of Prof. Running's monograph. The author summarises the most useful types of laws that are reducible to straight-line laws by means of simple transformations. He also gives practical rules for deciding whether such a law is correct for the given data and for the determination of the constants. Nineteen laws are discussed, and illustrated by means of numerical examples, whilst curves are drawn to show graphically the types of relations given by these laws.

A twentieth law is the Fourier expansion. Chapters are added on the method of least squares, interpolation, and numerical integration. The result is an eminently useful handbook for the scientific researcher and the practical engineer, and a highly commendable adjunct to the more theoretical study of mathematics.

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The pedagogy is, however, somewhat defective. It is difficult to imagine such a book in the hands of a student. The philosophy of the subject is scarcely entered into at all, and in places, where an attempt at justification is made, the result is not satisfactory. Also, one question remains unanswered: How is one to guess which law to try? Is one to try them all one after the other until the right one is reached? And what if none of those given is correct? Information on this and other points is very desirable, but none is offered.

One or two definite criticisms must be made. The different schemes in the chapter on Fourier series are not always consistent, and some are incomplete. No explanation is given of the meaning of "weights" in the method of least squares. In the chapter on interpolation the difference formula is proved only for integral values of the argument, and then applied to fractional values. In addition, there are a few misprints and some evidences of carelessness. The book thoroughly deserves a second edition, in which, it is hoped, these and other faults will be rectified.

(2) This is a very good book on the calculus, written in the old style with which we have been familiarised by writers like Edwards and Williamson. It is very well written and compact in form; the diagrams are good, and the exercises excellent. Particular attention is paid to questions of a practical nature. The student who has worked through this book conscientiously will have a good, if dull, appreciation of the subject and its manipulation. A few of the pages are headed "Unconventional Methods," but the thrill one gets on seeing this only leads to disappointment. There is nothing unconventional in an involute or in a parabola rotating about an axis.

The second part (which is also issued separately) includes the usual chapters on differential equations and the usual box of tricks.

This book, like so many others, gives the student the impression that there is just one particular integral of a linear differential equation in which the right-hand side is a function of the independent variable. It is more useful to inform the student that there are, of course, an infinite number of particular integrals, but that one of them is obtainable most readily and directly.

In Ex. 3, p. 10, of the second part it would have been more reasonable to put a negative sign to indicate the retarding effect of friction.

(3) Dr. Milne and Mr. Westcott have given expression to an important and fundamental principle in mathematical pedagogy—namely, the secondary nature of the manipulative art, and the first-rate importance of the ideas and methods of mathematics. They have recognised that the main part of the essence of the calculus, and even the most important practical applications of its processes, can be taught and learnt without using anything but the simplest of all functional types—namely, x^n and combinations of powers of x . When once the student has learnt to differentiate x^n , he is ready for much of the mysterious dis-

cipline that constitutes the black art of the calculus method. He can do dynamical problems without the aid of confusing formulæ; he can measure the volume of a tree; he can enclose land economically; he can draw tangents and normals; he can find radii of curvature; he can even solve differential equations. The authors, having recognised this fact, have acted upon it boldly and frankly, with the result that they have produced a book of a peculiarly suggestive and persuasive kind. Both authors are experienced teachers of mathematics, and the practical touch introduced by the physical propensities of one of them is everywhere noticeable. It is also refreshing to see $dy/dx=f(x)$ treated as a differential equation.

The merit of the book is somewhat marred by a few faults, and especially by the mediocre diagrams. Some are not well produced, whilst others are not even well drawn. The authors, or their artistic representative, seem to have an un-failing belief that a circle in perspective can be represented by two circular arcs intersecting at sharp angles. This is a gratuitous trap for the unwary.

The style is splendid. The preface is worth reading for its own sake, whilst the historical sketch with Isaac Barrow's prayer will interest even such students as are not excited by Guldinus's and Pappus's theorems.

(4) Many teachers have experienced the want of books on mechanics more advanced than the easy text-books used in schools, and not so advanced as the larger treatises intended for specialists in mathematics. Mr. Fawdry's books are supplying this want, and the present volume is a further contribution to the author's series of books on mechanics. This volume forms the second part of his "Dynamics," and discusses such subjects as differentiation and integration as used in dynamics, harmonic motion, and easy two-dimensional rigid dynamics. The work is well done. The experimental hints, the numerical illustrations of dynamical laws and results, and the very practical examples all help to make the subject attractive and intelligible. There is some lack of logic in the arrangement, and the impression one gets is that of scrappiness. The chapter on harmonic motion, *e.g.*, seems out of place in the middle of a discussion of rigid dynamics.

Mr. Fawdry wastes time in proving that the acceleration d^2x/dt^2 can be written vdv/dx . Surely it must be a part of fundamental dynamical doctrine that:

Number of units of force = time rate of momentum;

Number of units of force = space rate of kinetic energy.

This saves much trouble and memory-searching. One cannot feel angry with a student who forgets the trick of "multiplying by twice the velocity" to get the energy equation.

The figure on p. 271 is unfortunate: when a spiral spring is stretched, the pitch is increased.

One can heartily recommend this as a sound

book that will be found very useful both in itself and as an introduction to the larger treatises on the subject.

(5) The fact that a new edition is called for of Prof. Heath's "Solid Geometry" proves that it has been found to serve its purpose as an introduction to those parts of the subject that are required for their practical usefulness. The book, while making no pretence to pedagogical originality, is a very good collection of the most useful theorems and problems in solid geometry. It includes the geometry of the regular solids, spherical geometry, and the mensuration of the sphere. There are a large number of examples with some hints for their solution.

S. BRODETSKY.

VAGUENESS AND DISCRIMINATION.

(1) *The Intuitive Basis of Knowledge. An Epistemological Inquiry.* By Prof. N. O. Lossky. Authorised translation by Nathalie A. Duddington. With a preface by Prof. G. Dawes Hicks. Pp. xxix + 420. (London: Macmillan and Co., Ltd., 1919.) Price 16s. net.

(2) *Cultural Reality.* By Dr. Florian Znaniecki. Pp. xv + 359. (Chicago: The University of Chicago Press; London: Cambridge University Press, 1919.) Price 2.50 dollars net.

IT is extraordinary how difficult it seems to be (and how fearfully long the argument is) to convince a man that what he is quite ready to believe, until you make him doubt it, is true. Natural realism—the theory that the objects of knowledge are in themselves what they are represented to be in our knowledge, that knowledge is the discrimination by the mind of a reality awaiting discrimination—is, I suppose, the philosophical theory of knowledge we all hold until we are philosophers consciously philosophising. Tables and chairs are just tables and chairs, and would be such, so far as their essential form and matter are concerned, were there no mind, or, as the realist prefers to say, were there no act of discrimination, in the universe. We all believe it, but let us once challenge a realist philosopher to prove it—he may be able to, but, unlike the Rabbi called on to expound the whole of the law and the prophets, not while you stand on one foot.

(1) Prof. Lossky's "Intuitive Basis of Knowledge" is admirably translated and very clear and easy to read. The translator, Mrs. Duddington, is eminently qualified for the work, not merely by her knowledge of the original language, but also by what is far more important, her complete sympathy with the philosophical view of the author. The book is prefaced by a particularly lucid "Introduction" by Prof. G. Dawes Hicks, who, though not in entire agreement with the author, is very sympathetic towards his point of view. Prof. Hicks expresses surprise that a professor in a Russian university should have reached conclusions so strikingly in accord with his own, but, though Petrograd may be a long way from London, it is no further from